

Electron counting with a two-particle emitter

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We consider two driven cavities (capacitors) connected in series via an edge state. The cavities are driven such that they emit an electron and a hole in each cycle. Depending on the phase lag the second cavity can effectively absorb the carriers emitted by the first cavity and nullify the total current or the setup can be made to work as a two-particle emitter. We examine the precision with which the current can be nullified and with which the second cavity effectively counts the particles emitted by the first one. To achieve single-particle detection we examine pulsed cavities.

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I. INTRODUCTION

The dynamics of a quantum coherent capacitor connected via a single contact to an electron reservoir have attracted experimental and theoretical interest. A capacitor connected via a quantum point contact (QPC) to an edge state shows mesoscopic capacitance oscillations and a quantized charge relaxation resistance.¹⁻⁵ In addition a recent experiment demonstrated an "electron gun" emitting and absorbing a single electron in every oscillation cycle.⁶ The emission process⁷⁻⁹ injects an electron into states above the Fermi level, whereas absorption of an electron leaves a hole below the Fermi energy. The invention of lasers revolutionized optics. Similarly, single-electron injectors either using capacitors or quantized electron pumps¹⁰⁻¹⁴ provide novel coherent sources for electronics.

It is a challenging task to detect the electrons with the speed they were emitted with. In modern experiments the dynamics of single-electron transport through a mesoscopic system is often explored experimentally using as a charge detector, either a radio-frequency single-electron transistor¹⁵⁻¹⁷ or a QPC.¹⁸⁻²¹ However the speed of these detectors is not sufficient to detect electrons with a nanosecond resolution. To circumvent this problem, we propose as a fast detector a device which is analogous to the emitter: a quantum capacitor, such as used in Refs. 2 and 6. Such a detector is able to register particles as fast as an emitter can inject them into the quantum circuit. We therefore consider a system consisting of two quantum cavities coupled in series by a single edge state and modulated by in general different periodically varying potentials, both with frequency Ω , as shown in Fig. 1.

The charge emitted by the first cavity is detected by nullifying the *total* current with the use of the modulation of the second cavity. Namely, the potential $U_2(t)$ can be chosen in such a way that the total current vanishes. In general the current $I(t)$ consists of a series of pulses corresponding to electrons and holes emitted by either of the cavities. However if the time when an electron was emitted by the first cavity coincides with the time when a hole was emitted by the second cavity the total current $I(t)$ is suppressed. This electron-hole annihilation process can be viewed as the reabsorption by the second cavity of an electron emitted by the

first cavity and it can be used to count electrons. If the counting efficiency is perfect the total current vanishes completely.²²

Since the capacitor system generates an ac current it is convenient to investigate the degree of the current suppression by studying the square of the current integrated over one period $2\pi/\Omega$,

$$\langle I^2 \rangle = \int_0^{2\pi/\Omega} dt I^2(t). \quad (1)$$

Note that $\langle I(t)^2 \rangle$ is different from the noise (see Refs. 7-9). We develop the conditions for nullifying the total current and investigate the measuring precision. Alternatively, being driven in phase such a double-capacitor system can serve as a two-electron (two-hole) emitter.

II. MODEL AND FORMALISM

The system consists of two cavities with edge states of circumference L_1 and L_2 connected via QPCs with the reflection (transmission) amplitudes $r_1(t_1)$ and $r_2(t_2)$ to an edge state of length d and modulated by time-dependent potentials $U_1(t)$ and $U_2(t)$, respectively. A particle with energy E entering the cavity j picks up a phase kL_j , which is the kinetic phase of the guiding center motion.²³ The time τ_j that a particle spends for one revolution in the cavity j is related to the cavity's level spacing, $\Delta_j = \hbar/\tau_j$. Due to the time-dependent potential $U_j(t)$ an additional time-dependent phase $\Phi_j^q(t) = \frac{e}{\hbar} \int_{t-q\tau_j}^t dt' U_j(t')$ is accumulated in the cavity during q revolutions.

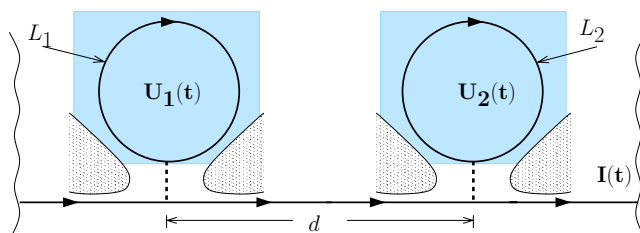


FIG. 1. (Color online) Two driven cavities (mesoscopic capacitors), formed with quantum point contacts, are coupled in series by an edge state. Time-dependent potentials $U_1(t)$ and $U_2(t)$ act homogeneously on the regions of the two cavities.

lutions. The separate cavities can be described by a time-dependent scattering matrix for a particle with incoming energy E , leaving the system at time t , given by the Fabry-Pérot-type expression,

$$S_j(E, t) = r_j + t_j^2 \sum_{q=1}^{\infty} r_j^{q-1} e^{iqkL_j - i\Phi_j^q(t)}. \quad (2)$$

With the time τ_d which the particle spends in the connecting edge state, the scattering matrix of the full system is

$$S_{\text{tot}}(t, E) = \sum_{p, q=0}^{\infty} ((r_2^*)^{-1} \delta_{p0} + t_2^2 r_2^{p-1} e^{ipkL_2 - i\Phi_2^p(t)}) \times ((r_1^*)^{-1} \delta_{q0} + t_1^2 r_1^{q-1} e^{iqkL_1 - i\Phi_1^q(t - \tau_d - p\tau_2)}) e^{ikd}. \quad (3)$$

A Floquet scattering matrix approach,^{7–9} used to deal with quantum pumping,^{24–29} enables us to investigate the dynamics of the system beyond the linear-response regime and adiabatic approximations. The full time-dependent current response to a periodic modulation with frequency Ω is

$$I(t) = \frac{e}{h} \int dE \sum_{n=-\infty}^{\infty} [f(E) - f(E + \hbar n\Omega)] \times \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt' e^{in\Omega(t-t')} S_{\text{tot}}^*(t', E) S_{\text{tot}}(t, E). \quad (4)$$

In the following we analyze the conditions to achieve efficient particle counting in the double-capacitor system by nullifying the total current and discuss the precision.

III. RESULTS

Inserting the total scattering matrix, given in Eq. (3) into the current formula of Eq. (4), we obtain a general result for the total current due to a harmonic modulation of the system. We first investigate the adiabatic regime, specifying results at zero and at high temperatures. Subsequently corrections to the adiabatic results and the strongly nonadiabatic limit are considered.

A. Adiabatic response

In the following we calculate the current response to two potentials $U_j(t) = \bar{U}_j + \delta U_j(t)$. In the adiabatic limit, $\Omega \rightarrow 0$, where the time scale set by the modulation is much larger than the time particles spend in the cavities and the connecting edge state, we expand Eq. (4) in first order Ω . The current $I^{(1)}(t)$ is related to the instantaneous densities of states $\nu_j = \nu_j(t, E) = \frac{1}{2\pi i} S_j^*(E - eU_j(t)) \frac{\partial S_j(E - eU_j(t))}{\partial E}$ of the two cavities,

$$I^{(1)}(t) = e^2 \int dE [-f'(E)] \left[\nu_1 \frac{\partial U_1(t)}{\partial t} + \nu_2 \frac{\partial U_2(t)}{\partial t} \right]. \quad (5)$$

With the transmission $T_j = |t_j|^2$ the density of states is

$$\nu_j(t, E) = \frac{1}{\Delta_j 2 - T_j - 2\sqrt{1 - T_j} \cos \phi_j(E, t)}. \quad (6)$$

The phase $\phi_j(E, t)$ can be written as the sum of a time-dependent and a time-independent contribution,

$$\phi_j(E, t) = -2\pi e \delta U_j(t) / \Delta_j + 2\pi \chi_j(E), \quad (7)$$

with $2\pi \chi_j(E) = \frac{\tau_j}{\hbar} (E - \mu) + k(\mu) L_j - 2\pi e \bar{U}_j / \Delta_j + \phi_j^r$. The phase of the respective QPC's reflection coefficient is given by ϕ_j^r , with $r_j = |r_j| \exp(i\phi_j^r)$. The detuning $\chi_j \Delta_j$ defines the position of the quantum level in the cavity j with respect to the Fermi level μ at zero driving amplitude, $\delta U_j(t) = 0$. Thus to lowest order in frequency the current consists of a sum of separate contributions of the two cavities. This means that the total time-dependent current can be nullified whenever the phase difference $\delta_2 - \delta_1$ of the two harmonic modulations is equal to π and when the amplitudes together with the cavity parameters are adjusted in an appropriate way. We now choose a harmonic time dependence, $\delta U_j(t) = U_j \cos(\Omega t + \delta_j)$, with $U_j > 0$.

B. Current nullification at $k_B T = 0$

In Fig. 2 we plot the time integral of the squared current [Eq. (5)] as a function of the phase difference of the potentials given for different choices of detuning $\chi_1 \Delta_1$, $\chi_2 \Delta_2$ and for different values for eU_2 / Δ_2 . To understand these plots, we consider the limit of very low transmission at the QPCs, $T_j \ll 1$. Then the instantaneous density of states [Eq. (6)] of the cavities takes the form of a sum of Breit-Wigner resonances, around the zeros of the quantity $\phi_j(E, t)$ —defined in Eq. (7)—to be taken mod 2π . We take the modulation amplitude U_j to be smaller than half the level spacing Δ_j and larger than the detuning $\chi_j \Delta_j$, such that one electron and one hole are emitted per cycle. We consider particles with energies equal to the Fermi energy.

When periodically driving the potentials $U_j(t)$, the densities of states have a peak at the Fermi energy around resonance times t_j^+ and t_j^- . To lowest order Ω , the current pulse generated [Eq. (5)] is expressed in terms of the resonance times, t_j^\pm , and the half-widths of the pulses, w_j ,

$$\Omega t_j^\pm = -\delta_j \pm \arccos\left(\frac{\chi_j \Delta_j}{eU_j}\right), \quad (8a)$$

$$\Omega w_j = \frac{1}{2\pi} \frac{T_j \Delta_j}{2eU_j} \left[1 - \left(\frac{\chi_j \Delta_j}{eU_j}\right)^2 \right]^{-1/2}. \quad (8b)$$

We are interested in a situation where during the driving process an electron and a hole are fully emitted, separately from each other, and therefore the distance between the resonance times t_j^\pm is much larger than the width of the current pulse, $|t_j^+ - t_j^-| \gg w_j$. We find

$$\langle (I^{(1)})^2 \rangle = \frac{e^2}{\pi} \left[\frac{1}{w_1} + \frac{1}{w_2} \right] + \frac{2e^2}{\pi} [L(t_1^+ - t_2^+) + L(t_1^- - t_2^-) - L(t_1^+ - t_2^-) - L(t_1^- - t_2^+)], \quad (9)$$

where we introduce the Lorentzian $L(X) = (w_1 + w_2) / [X^2 + (w_1 + w_2)^2]$. Its arguments $(t_1^\pm - t_2^\pm)$ are taken mod 2π in the interval $[-\pi/\Omega, \pi/\Omega]$. The four Lorentzians contribute only if the respective resonance times are close to each other compared to the width of the current pulse. If the first two Lorentzians contribute, two particles are emitted by the sys-

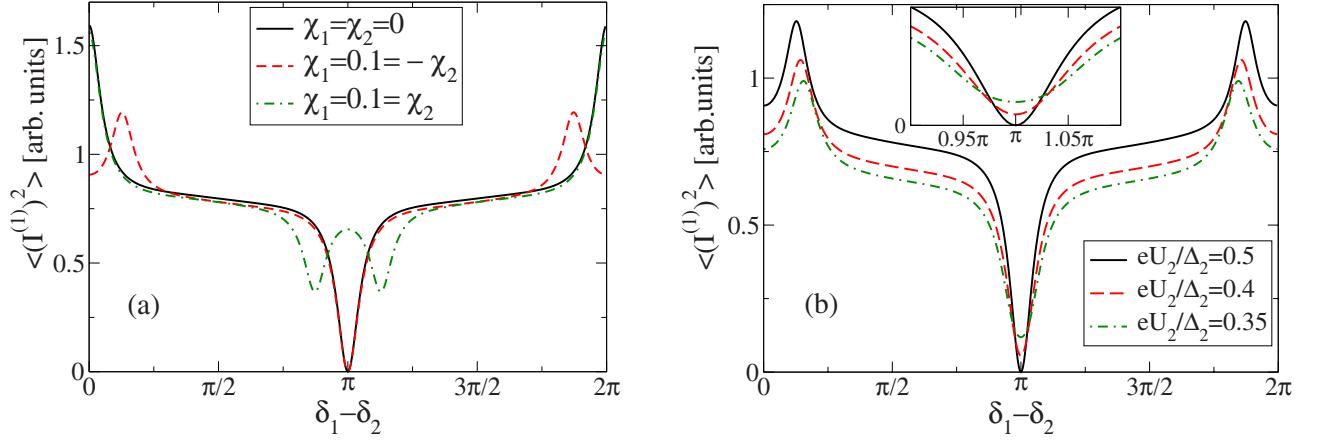


FIG. 2. (Color online) Zero-temperature limit of the averaged square current $\langle (I^{(1)})^2 \rangle$ from Eq. (5) as a function of the phase difference of the modulation potentials. The ratio between modulation amplitude and the level spacing of the first cavity is given by $eU_1/\Delta_1 = 0.5$. The transmission probabilities of the QPCs of the two cavities are $T_1 = T_2 = 0.4$. (a) Different detuning and $eU_2/\Delta_2 = 0.5 = eU_1/\Delta_1$. (b) Fixed detuning $\chi_1 = 0.1 = -\chi_2$ and different values of eU_2/Δ_2 .

tem at the same time either two electrons or two holes and $\langle (I^{(1)})^2 \rangle$ is maximized. We are instead interested in the situation where both of the last two terms contribute, meaning that one cavity emits a hole approximately at the same time as the other emits an electron and vice versa. The conditions for nullifying the current exactly are

$$\delta_1 - \delta_2 = \pi, \quad (10a)$$

$$\chi_1/T_1 = -\chi_2/T_2, \quad (10b)$$

$$eU_1/(T_1\Delta_1) = eU_2/(T_2\Delta_2). \quad (10c)$$

Experimentally these conditions can be obtained by tuning the phase χ_j , the amplitude of the time-dependent part, and the phase difference of the potentials. Close to these conditions, $\langle (I^{(1)})^2 \rangle$ as a function of the phase difference has a pronounced dip,

$$\langle (I^{(1)})^2 \rangle = \frac{2e^2}{w\pi} \frac{(\delta_1 - \delta_2 - \pi)^2}{(\delta_1 - \delta_2 - \pi)^2 + 4w^2\Omega^2}, \quad (11)$$

where $(\delta_1 - \delta_2 - \pi)$ is taken mod 2π on the interval $[-\pi, \pi]$. In Fig. 2(a) we show $\langle (I^{(1)})^2 \rangle$ as a function of $\delta_1 - \delta_2$ at finite transmission probability of the QPCs for $\frac{eU_1}{T_1\Delta_1} = \frac{eU_2}{T_2\Delta_2}$. Whenever the maximum is at $\delta_1 - \delta_2 = 0$, both the two electrons and the two holes are emitted at the same time (solid and dashed-dotted curves, respectively). Whenever the minimum is at $\delta_1 - \delta_2 = \pi$, any emitted electrons are annihilated by holes at the same time (solid and dashed lines). If the current pulses of an electron of one cavity and a hole of the other are both coinciding but the widths of the pulses are different, the distance of the minimum of $\langle (I^{(1)})^2 \rangle$ from zero is

$$\langle (I^{(1)})^2 \rangle = \frac{e^2}{\pi} \frac{(w_1 - w_2)^2}{w_1 w_2 (w_1 + w_2)}, \quad (12)$$

showing a smooth dependence on the system parameters. It guarantees the robustness of the dip against small deviations from the ideal conditions. This minimum for the more general case of Eq. (5) is shown in Fig. 2(b).

C. High temperatures, $k_B T \gg \Delta_j$

In this regime the quantized emission is destroyed. However the current nullification can still be achieved and, e.g., be used to tune the parameters of the cavities. At high temperatures we use $\nu_j = 1/\Delta_j$ in Eq. (5). Then from Eq. (1) we find that the time integral of the square of the low-frequency current takes a particularly simple form,

$$\frac{\langle (I^{(1)})^2 \rangle}{\pi e^2 \Omega} = e^2 \left(\frac{U_1^2}{\Delta_1^2} + 2 \frac{U_1 U_2}{\Delta_1 \Delta_2} \cos(\delta_1 - \delta_2) + \frac{U_2^2}{\Delta_2^2} \right). \quad (13)$$

It shows a cosinelike behavior as a function of the phase difference, in contrast to the zero-temperature result, where the width of the dips (peaks) is determined by w_j . Independently of the detuning of the two cavities and the transmission of the QPCs, $\langle (I^{(1)})^2 \rangle$ is exactly zero when $eU_1/\Delta_1 = eU_2/\Delta_2$ and $\delta_1 - \delta_2 = \pi$ and deviates from zero at $\delta_1 - \delta_2 = \pi$ by $\pi e^2 \Omega (eU_1/\Delta_1 - eU_2/\Delta_2)^2$.

D. Correction to the adiabatic response

The response in second order in frequency

$$I^{(2)}(t) = -\frac{e^2 \hbar}{2} \int dE [-f'(E)] \frac{\partial}{\partial t} \left[\nu_2^2 \frac{\partial U_2(t)}{\partial t} + \frac{\partial U_1(t)}{\partial t} (\nu_1^2 + 2\nu_1 \nu_2 + 2\nu_1 \nu_d) \right], \quad (14)$$

contains mixed terms in the densities of states of the cavities and the connecting channel, $\nu_d = \nu_d(E)$, as well. Comprising information about the entire system, it can lead to nonvanishing contributions in the regime where the adiabatic current response vanishes. It is interesting to consider corrections in higher order Ω [Eq. (14)], which when $\langle (I^{(1)})^2 \rangle$ vanishes are dominant. Independently of the temperature regime, the correction to $\langle I^2 \rangle$ in second order in Ω , for the parameters where $\langle (I^{(1)})^2 \rangle$ in first order in Ω vanishes, is always zero. The leading term in Ω of $\langle I^2 \rangle$ is then at least of third power in Ω .

At zero temperature and small QPC transmission under the conditions given in Eq. (10) we find that $\langle I^2 \rangle \sim \langle (I^{(2)})^2 \rangle$ is of the order $\langle (I^{(2)})^2 \rangle \sim (e^2/w)(\tau_1/Tw)^2$, where τ_1/T is the dwell time for an electron in the first cavity. In comparison, the contribution stemming from the first order in frequency current away from resonance is $\langle (I^{(1)})^2 \rangle \sim e^2/w$ [see Eq. (9)]. We find that in the nonlinear regime the adiabaticity condition is $I^{(2)}/I^{(1)} \sim \Omega\tau/T^2 \ll 1$, which differs strongly from the one in linear response, $\Omega\tau/T \ll 1$.

At high temperature and at the parameters nullifying Eq. (13), $\langle (I^{(2)})^2 \rangle$ is in general not zero and it depends additionally on the transmission of the QPCs and the inverse of the density of states of the connecting edge state, Δ_d . However, in contrast to the low-temperature regime $\langle (I^{(2)})^2 \rangle$ can be nullified by introducing further transmission-dependent conditions. These conditions are directly obtained from Eq. (14) and read for cavities of unequal lengths, $\frac{2-T_1}{2T_1} + \frac{\Delta_1}{\Delta_d} - \frac{3T_2-2}{2T_2} \frac{\Delta_1}{\Delta_2} = 0$.

E. Nonadiabatic step-potential modulation

In an experimental setup, instead of by a sinusoidal modulation, the system is often driven by a square-pulse potential, where a treatment in the highly nonadiabatic regime is required. For the following analysis we start from Eqs. (3) and (4) and limit ourselves to the high-temperature regime. We are interested in a step potential which is the limit of a periodic square-pulse modulation with infinitely long period. In principle, the potential at the two cavities can have different amplitudes and can be switched on at different times t_1^0 and t_2^0 , with $t_2^0 = t_1^0 + \tau_d + \Delta t^0$, the sum of the switching time of the first cavity t_1^0 , the time a particle needs to pass through the connecting edge state τ_d , and a time delay Δt^0 , where here we choose $\Delta t^0 = 0$. The step potentials at the two cavities read $U_j(t) = U_j$ if $t \geq t_j^0$ and 0 otherwise. The cavities' response to the potentials decays with a characteristic time given by the bigger value of $\{\tau_1/\ln(1/R_1), \tau_2/\ln(1/R_2)\}$. After a waiting time which is much bigger than the decay time, the charge emitted by the system equals the sum of the charges that would be emitted by two completely independent cavities and is given by $Q = e \frac{eU_1}{\Delta_1} + e \frac{eU_2}{\Delta_2}$. While this charge is nullified for $\frac{eU_1}{\Delta_1} = -\frac{eU_2}{\Delta_2}$, the nullifying of the integral of the squared current can in general not be reached, meaning that an ac current is generated.

To find some simple analytical results, let us restrict ourselves to the limit of identical ($r_1 = r_2$ and $L_1 = L_2$) weakly coupled cavities ($T_1 = T_2 = T \rightarrow 0$) and consider the interesting case where the total charge is nullified and $\langle I^2 \rangle$ is suppressed, i.e., $U_1 = -U_2 \equiv U$. For a single cavity as well as for the double-cavity system we find the time integral over the squared current to be of the form

$$\langle I^2 \rangle = \frac{e^2 (eU)^2}{h \Delta} F(T, U). \quad (15)$$

The function F for a system with a single cavity is given by $F_{\text{single}} = T/2$. For the double-cavity system with equal lengths, $F(T, U)$ oscillates in the potential difference with a phase $\phi = 2\pi e(U_1 - U_2)/\Delta = 4\pi eU/\Delta$. We find $F_{\text{double}} = T^3/[2 - 2\cos(\phi)]$ if $\phi \neq 2n\pi$, $F_{\text{double}} = T^3/2$ if $\phi = (2n+1)\pi$, and $F_{\text{double}} = T/4$ if $\phi = 2n\pi$. The time integral of the squared current is of the same order for the system of a single and a double cavity, showing that the coupling between the two cavities is important in the highly nonadiabatic regime. This is indicated already in Eq. (14), where in second order Ω mixed terms in the densities of states of the two cavities appear.

IV. CONCLUSIONS

We investigated the ac current response of a two-particle emitter consisting of a double-cavity system and propose it as an efficient tool for counting electrons emitted at high speed. The square of the total current integrated over one period shows a pronounced dip when the two cavities are synchronized. We extract the conditions for perfect counting by complete current nullification and show that in the adiabatic regime the counting efficiency is maintained at small deviations from the obtained conditions. In the highly nonadiabatic regime, current nullification can in general not be obtained. However, in principle, pulsed cavities can be used to analyze single events.

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